**Operations Analytics Take Home Assignment  
BEM3062**

**Group 3**

1. The break-even point is determined using the following formula:

|  |
| --- |
|  |

This calculation allows us to identify the quantity of units that must be sold for total revenue to equal total costs, leading to neither profit nor loss.

The Goal Seek method is particularly effective for this purpose as it automates the process of solving equations with one variable. It allows you to change the value of a single input cell to reach a desired output in another cell

We start by building a simple model that links profit with sales volume. The relationship between profit, revenue, and costs is defined by the formula:

|  |
| --- |
|  |

Here, Total Revenue is calculated by multiplying the number of umbrellas sold by the selling price per umbrella [B10\*B6], while Total Costs are the sum of fixed costs and total variable costs [B4+B5\*B10]. To find the break-even point where profit equals zero, use Goal Seek:

A screenshot of a spreadsheet

Description automatically generated

In the above scenario, the input cell is the "Number of Umbrellas Sold" in cell B10, while the target output cell is "Profit” in cell B15. By setting the profit to 0, Excel determines the exact quantity of units required to sell so that total revenue covers both fixed and variable costs, achieving break-even.

A screenshot of a spreadsheet

Description automatically generated

With the Goal Seek method applied in Excel, it has calculated that 20,000 umbrellas need to be sold for Ultra Company to break even.

Using the Data Table function from the What-If Analysis in Excel, I can calculate how different quantities sold will affect profit.

A screenshot of a spreadsheet

Description automatically generated

1. The relationship between quantity sold and profit in this table shows a direct correlation: as more units are sold, the fixed costs are spread across a larger number of units, and the revenue from sales starts to exceed total costs. Initially, losses decrease with each increase in quantity sold due to the rising contribution margin (the difference between selling price and variable cost). After surpassing the break-even point at 20,000 units, any further sales result in net profit, demonstrating how sales volume drives profitability.
2. The two-way table shows that both sales volume and variable costs significantly impact profitability. Higher sales volumes generally improve profitability, but this positive effect can be offset if variable costs rise too much. It highlights the importance of managing production costs and achieving sufficient sales volume to ensure profitability, emphasizing that both factors must be balanced for optimal financial performance.

A screenshot of a spreadsheet

Description automatically generated

A graph with a line going up

Description automatically generated

A screenshot of a spreadsheet

Description automatically generated

1. The decision variables are:

x = Number of days Mine X operates per week  
y = Number of days Mine Y operates per week  
  
Justification:  
The decision variables represent controllable quantities we can control to meet the ore production requirements at minimum cost.   
  
Assumptions for mine operation:

* 5 days/week (exclude weekends).
* Ore production is consistent and unaffected by external factors.
* The decision variables have a linear relationships, allowing for the use of linear programming for optimisation.
* The decision variables have non-negativity and it is because you have a negative number of days to operate the mines.

1. High-grade ore constraint: 6x+y ≥ 12   
   The combined weekly production from Mine X (6 tons/day) and Mine Y (1 ton/day) must be at least 12 tons.

Medium-grade ore constraint: 3x+y ≥ 8  
The combined weekly production from Mine X (3 tons/day) and Mine Y (1 ton/day) must be at least 8 tons.  
  
Low-grade ore constraint: 4x+6y ≥ 24  
The combined weekly production from Mine X (4 tons/day) and Mine Y (6 ton/day) must be at least 24 tons.  
  
Operating days constraints: x ≤ 5 and y ≤ 5  
Each mine can operate up to 5 days due to weekend closures

Non-negativity constraints: x ≥ 0 and y ≥ 0   
Minimum operating days must be 0.

Assumptions:  
Daily production rates are fixed and no ore is wasted.

**Analysis Step-by-Step:**

1. Step 1:

Objective: Minimise total weekly operating cost while meeting production requirements for high, medium and low-grade ore.

Objective function:

Total weekly cost Z = 180x + 160y (£000s)

The firm seeks to minimise cost by varying x and y, assuming costs are linearly proportional to operating days

1. Step 2:

Rearrange constraints into the form **y=mx+c**

|  |  |
| --- | --- |
| High-grade ore | y≥12-6x |
| Medium-grade ore | y≥8-3x |
| Low-grade ore | y≥ 4-(2/3)x |
| Operating days for Mine X & Y | x≥0; y≥0; x≤5; y≤ |

Step 3:

Z= 180x+160y  
Rearranging for y:  
y= (Z/160) - (9/8)x

Step 4:

Find the x y crossing points for the constraints  
**High-grade ore:** 6x+y=12   
Setting y=0, x=2  
Setting x=0: y=12

**Medium-grade ore:** 3x+y=8  
Setting y=0, x=8/3  
Setting x=0, y=8

**Low-grade ore:** 4x+6y=24  
Setting y=0, x=6  
Setting x=0, y=4

Step 5:

Plot the constraints:

A graph of a function

Description automatically generated

Step 6

Identify feasible domain for each line  
Pick point (1, 1) & substitute in each equation:

y≥12-6x

1 ≥ 6. Region below line infeasible

y≥8-3x

1 ≥ 5. Region below line infeasible.

y≥ 4-(2/3)x

1 ≥ 10/3. Region below line infeasible

x ≥ 0

1 ≥ 0. Region to left of line is infeasible  
y ≥ 0

1 ≥ 0. Region below line is infeasible  
x ≤ 5

1 ≤ 5. Region to right of line is infeasible  
y ≤ 5

1 ≤ 5. Region above line is infeasible

The optimal solution (grey) is below:

A diagram of a function

Description automatically generated

Step 7:

Objective function is:

y=(Z/160)-(9/8)x

Pick random value of Z=480 and get objective fn line & crossings:

y=3-(9/8)x

At y=0, x=8/3

At x=0, y=3

Step 8/9:

Plot objective line and lines parallel to it

A diagram of a magic point

Description automatically generated

Step 10:

Get the magic point at the intersection of a parallel line and the region crossing tangent/apex. This also crosses at

y=8-3x

y=4-(2/3)x

Step 11:

Calculating, the magical point is x=12/7 y = 20/7

Step 12:

Calculate optimal solution:

Substitute (12/7, 20/7) into Z=180x+160y  
Z=180(12/7)+160(20/7)

Z=765.714  
  
Now multiply Z by 1000 to get correct optimal cost, £765,714 and operate Mine X for 1.71 days and Mine Y for 2.86 days.

1. The decision faced here is how many toy soldiers and trains to produce to maximize profit.

Decision Variables:

x= number of soldiers to produce

y= number of trains to produce

Assumptions made:

* The decision is a *linear problem* which assumes that the two decision variables and the objective function (total profit) have a linear relationship. For example, for every additional unit of ‘soldiers’ produced, profit should increase at a proportionate rate. The constraints involved also have a linear relationship. This allows the use of linear programming for optimization.
* The decision variables are *non-negative*: This is because you cannot produce a negative number of a product.
* *Resource Scarcity* ensures that the constraints are binding which ensures all constraints are inequalities, which is essentially a bottleneck that prevents net profit from increasing past the optimal amount calculated through the model.
* The maximum profit is *unbounded*. This means the limit is infinity.

1. Constraints (Availability of hours is on a weekly basis):

* Finishing Labor Constraint: 2x+y<=100. Here, each soldier needs 2 hours, and each train takes 1 hour of finishing but only 100 hours are available each week for finishing
* Carpentry Labor Constraint: x+y<=80. Here, each soldier needs 1 hour, and each train needs 1 hour of carpentry but only 80 hours are available each week for carpentry.
* Demand Constraint: x<= 40. Here, demand for trains is unlimited but only 40 soldiers can be sold each week.
* Non-negativity: x>=0 and y>=0

1. The objective function here is Max P=3x+2y. This is derived from the unit margins of toy soldiers and trains as calculated below.

A screenshot of a table

Description automatically generated

The assumptions made here are:

* Constant unit margins: No changing costs due to real-life factors such as economies of scale.
* No time value of money: The production cycle and the profit generated is one week. These often take time leading to a distortion in the above figures due to uncertainty and market forces (StudySmarter UK, n.d.).
* Linearity with the same logic as part A.

1. When profit is maximized, 20 soldiers and 60 trains should be produced. This has been derived has seen below.

A table with numbers and symbols

Description automatically generated

A screenshot of a computer

Description automatically generated

A graph paper with writing on it

Description automatically generated

A screenshot of a graph

Description automatically generated

1. If the selling price of wooden soldier is changed to £28 and the constraints and conditions are not changed, 40 soldiers and 20 trains should be produced to maximize profit. This is depicted below.

A graph on a piece of paper

Description automatically generated

Here, because profit is maximized and considering the slope of the new level line, (40,20) is the new magic point.

1. Consider the following graphical representation.Here there are no feasible solutions to the model, considering the infeasible inequalities, as the problem has been constrained.

A graph on a piece of paper

Description automatically generated

A table with numbers and symbols

Description automatically generated

A screenshot of a computer

Description automatically generated

Question 3 Excel File link: [GW BEM3062.xlsx](https://universityofexeteruk-my.sharepoint.com/:x:/g/personal/jjb249_exeter_ac_uk/ESlFufFgE91GgdB-W1HPXtMBARD2dW9j8kHUek-CFBOncQ?e=5ut1oX&nav=MTVfe0I4QjhCNERELTZGMDktMEE0Ni05MzA3LTZBRTk0MTdGRTc3NX0)

1. Linear Programming or Linear Optimization is a method for business to find optimal solutions of many cases, in terms of profit maximizing or lowest cost. Following the 7 steps with Rober Scott & Sons (RSS) in mind.

Step 1: we identify two decision variables: production of 400 logs and 600 Pallets a day.

* Therefore;
  + X = lumber produced a day
  + Y = number of pallets produced a day.

Step 2: Writing down the objective function.

* The cost for lumber is £200 per log, and a pallet cost £4 per ONE log.
* The processing cost of log to lumber is £200, and a log to pallets cost £5.
* Accordingly, it takes 1.4 logs to create ONE high-grade lumber and 0.25 logs to create ONE number of pallets.
* £200 \* 1.4 (logs needed to create 1 HG Lumber) = £280 + £200 (processing) = £480
* £4 \* 0.25 (logs needed to create 1 pallet) = £1 + £5 (processing) = £6
* The business sells the Lumber for £490 per mbf, and £9 for each pallet.
* X à £490 - £480 = £10|Y à £9 - £6 = £3

Profit Maximization(Z) = 10x + 3y

Step 3: identify the constraints.

* x1 ≤ 200 (lumber size capacity limited by its “Kiln”, an oven type)
* 1.4x + 0.25y ≤ 400 (maximum amount of logs a day)
* x3 ≤ 600 (maximum pallets produced a day)

Step 4: using DESMOS graph we can create the problem graphically using above.

Step 5: We constructed the graph using the sets of constraints.

Step 6: We identify the shaded region from the graph with the vertices

* X ≤ 200 (dried lumber per mbf)
* 1.4x + 0.25y ≤ 400 (logs per day)
* Y ≤ 600 (pallets per day)

Step 7: Using the vertices of the shaded area (or plot points) to determine the optimal solution.

Remember that profit maximization is à 10x + 3y = Z

* (178.57143, 600)
  + 10(178.57143) + 3(600) =3585.7143

We can conclude now that daily profit maximization is £3585.7143

1. Find the Optimal Solution.

Refer to steps 5–7 of the previous question.

1. Logs constraints is 1.4x + 0.25y ≤ 400
2. Lumber constraint is X ≤ 200
3. Pallet’s constraint is Y ≤ 600

Plot to graph, Vertices identified, remember that profit maximization is 10x + 3y ≤ 400

A graph paper with a blue line and red line

Description automatically generated

The graph above shows the optimal solution for Rober Scott & Sons Ltd’s optimal solution, to produce around (where the points meet [178.57, 600]):

* 178.57 Lumber per day
* And 600 pallets per day
* Which would account to making about £3585.7143 per day.

1. ([Answer problem for Question 4c-d.xlsx](https://universityofexeteruk-my.sharepoint.com/:x:/g/personal/at828_exeter_ac_uk/EYNs7dl8kbJHjOqTC4V1LgQBQVWGsRpZo5k2z4KWjrXgKQ?e=wnZ1K1)ß A hyperlink to the excel sheet

A screenshot of a spreadsheet

Description automatically generated

A screenshot of a computer

Description automatically generated

A screenshot of a computer screen

Description automatically generated

**References**

*Time Value of Money: Definition, Formula & Calculation*. (n.d.). StudySmarter UK. https://www.studysmarter.co.uk/explanations/business-studies/corporate-finance/time-value-of-money/

‌